

## **Low-Altitude Orbits for Space-Based Interceptors**

**P. B. Duffy  
C. T. Cunningham**

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## SUMMARY

This study has two goals: The first is to quantitatively evaluate several issues concerning basing SBIs at low altitudes in response to fast-acting ICBMs. These issues are

- How much is performance improved with low-altitude orbits?
- What air-drag penalty is incurred at low altitudes?
- Can elliptical orbits be used to reduce air drag?

The second goal is to use the same quantitative methods to find optimum orbit altitudes against various threats. Our approach to this problem is relatively narrow: we try to identify altitudes that get the best defense performance from a given amount of rocket propellant. In other words, we seek an optimum between very low orbits (which require large amounts of propellant for orbit maintenance) and higher orbits (which require less propellant for orbit maintenance but where SBIs need large amounts of propellant in order to reach their targets). Although very useful, this approach ignores some important considerations, chiefly defense survivability and SBI divert velocity requirements.

Our study has four main conclusions.

First, the performance of SBI defenses in general will not be improved much by reducing the basing altitude. Defense performance is much more sensitive to threat and defense action times and carrier vehicle (CV) orbit inclination than to the basing altitude. The main exception to this conclusion is when SBIs simply cannot reach their targets from higher altitudes.

Second, elliptical orbits do not save stationkeeping fuel compared to circular orbits, unless inclinations within a few degrees of  $63.4^\circ$  can be used. This is true because corrections for apsidal rotation, which are needed to keep the orbit properly oriented, consume more propellant than is saved by raising the altitude of apogee, which reduces drag. Defense performance is also very sensitive to orbit inclination (relative to target latitude), and hence the apsidal rotation problem for elliptical orbits cannot be avoided unless the defense attacks only targets very near  $60^\circ$  latitude. The conclusion that circular orbits are generally preferred over elliptical holds for a wide range of CV masses and aerodynamic characteristics and a wide range of solar activity levels.

Third, if SBIs have more than about 90 sec to reach their targets, altitudes above approximately 400 km are the most fuel efficient, and altitudes can be raised to 500 km or higher with very little decrease in effectiveness or increase in fuel consumption. The increased survivability presumably obtained at higher altitudes should be worth the cost.

Finally, SBI defenses are probably not practical if fast threats or long defense start-up delays reduce SBI flight times below about 50 sec. In this situation, conceivable options for the defense - either very low orbits or very high speed SBIs, or a very large number of SBIs - all require unreasonably large amounts of propellant.

## I. INTRODUCTION

A number of previous studies<sup>1-3</sup> have shown that an enemy's fast-acting ICBMs could significantly reduce the boost- and post-boost-phase effectiveness of SBI defenses. Faster-burning boosters or faster-acting busses or both could reduce the defense participation fraction (DPF), the fraction of SBIs that can reach at least one target in the available time.

One possible response to fast-acting ICBMs is to base SBIs in orbits much lower than the 500- to 700-km latitudes used in typical defense architectures.<sup>4</sup> Lower altitude orbits would reduce the distance SBIs must travel to reach their targets and thus increase the fraction of SBIs that can attack the threat.

However, there are penalties associated with low-altitude basing. Below about 400 to 500 km, satellites encounter significant air drag.<sup>5</sup> This resistance either causes the satellite to reenter prematurely, or forces energy to be expended to maintain the orbit. Because of the roughly exponential density profile of the atmosphere, air drag increases rapidly as altitudes are decreased.

In this study, we quantified the improved defense performance and the air-drag penalties associated with low orbits. We also attempted to find optimum orbital altitudes taking both of these factors into account.

Our approach is to try to get the best possible performance (i.e., to attack the most targets) with a given mass of rocket propellant. At high basing altitudes, very little stationkeeping propellant is required and propellant is used mainly to provide SBI velocity and range. As altitudes are lowered, more propellant is needed for

stationkeeping; this reduces the maximum velocity and range of the SBIs. Thus, as the CV altitude is lowered, two effects occur. The distance to the targets will decrease, which increases the DPF, but the maximum axial velocity of each SBI decreases, which has the opposite effect. By quantifying these effects, we find optimum orbit altitudes for various sets of assumptions about the threat and the defense.

We caution that this approach does not consider several issues relevant to SBI orbit selection. The most important of these issues may be the survivability of the CV platform. Against ground-based ASAT attack, survivability presumably increases with basing altitude, since this gives the defense more time to protect itself (by maneuvering, for example). There may even be a minimum basing altitude below which the defense cannot protect itself against ground-based ASATs, because the time for the ASAT to reach its target becomes less than the defense reaction time.

Another issue we have not analyzed is the dependence of SBI divert velocity requirements on orbit parameters. Recent work<sup>6</sup> with standard missile guidance algorithms (e.g., augmented proportional navigation) indicates that the divert velocity needed for an SBI to hit its target strongly depends on the engagement geometry (among other things). A particularly important parameter is the target's acceleration in the plane perpendicular to the line of sight of the SBI. (For example, a tail-chase or head-on engagement requires much less divert velocity and propellant than if the booster travels across the SBI's field of view.) This suggests that divert velocity requirements will depend on orbit inclination and altitude. We have not quantified this dependence, but assume that 2 km/sec of divert velocity is required in all cases. This is probably more than is needed for most typical engagements.



The issues of low-altitude orbit and propellant usage will be discussed as follows. In Section II, we estimate the propellant needed to maintain orbits, taking into account air drag and rotation of the line of apsides. In Section III, we explain our orbit optimization method: how we allot propellant mass between SBI propellant and fuel for orbit maintenance, as well as the assumptions and measures of merit we use to evaluate the effectiveness and fuel-efficiency of orbits. This section also gives our estimates of optimum altitudes against various idealized threats. In Section IV, we summarize our results.

## **II. PROPELLANT REQUIREMENTS FOR ORBIT MAINTENANCE**

Air drag can severely limit the lifetimes of satellites in very low orbits. Lifetimes depend on satellite characteristics and on the level of solar activity. Figure 1 shows that, under moderate solar conditions, lifetimes of a typical satellite can be as short as days or weeks. Clearly, the high cost of satellites makes it worthwhile to extend their lives by orbit maintenance.

The amount of energy and propellant needed to do this can be estimated by integrating the air drag force around the orbital path. The instantaneous drag force is<sup>5</sup>

$$D = \frac{1}{2} \rho v^2 C_d A \quad , \quad (1)$$

where  $\rho$  is the atmospheric mass density,  $v$  is the satellite velocity,  $A$  its frontal area, and  $C_d$  its coefficient of drag. Over a time interval  $dt$ , this force changes the satellite's velocity by an increment

$$dv = \frac{Ddt}{M} , \quad (2)$$

if  $M$  is the mass of the satellite. This is, therefore, the velocity increment that must be applied to the satellite to maintain its orbit. The mass  $dM$  of propellant required to do this is found from the single-stage rocket equation<sup>7</sup>:

$$dM = M \left[ 1 - \exp\left(\frac{-dv}{I_{sp}g}\right) \right] , \quad (3)$$

where  $I_{sp}$  is the specific impulse of the propellant and  $g$  is the acceleration of gravity. In the limit of frequently applied corrections ( $dv \ll I_{sp}g$ ), this reduces to

$$dM = \frac{Mdv}{I_{sp}g} . \quad (4)$$

The rate at which stationkeeping fuel is used is thus

$$\frac{dM}{dt} = \frac{M}{I_{sp}g} \times \frac{dv}{dt} = \frac{D}{I_{sp}g} . \quad (5)$$

For circular orbits, the fuel required per period is roughly

$$\Delta m = \frac{\pi r \rho v C_d A}{I_{sp}g} , \quad (6)$$

which is independent of satellite mass.

Air drag becomes a significant problem at altitudes below about 350 km. Figure 2 shows that at that altitude, about 100 kg of propellant per year is needed to maintain the orbit of a satellite with a 10-m<sup>2</sup> frontal area. (This assumes a drag coefficient of 2, the minimum possible in this density regime; atmospheric densities are taken from the 1976 U. S. Standard Atmosphere.<sup>8)</sup>)

The lowest altitude practical for long-term deployment, even if supported by a major national effort, is probably around 200 km. Figure 2 shows that at this altitude, a 10-m<sup>2</sup> satellite requires about 1000 kg of propellant per year to overcome drag. For 500 satellites, this would represent about 22 space shuttle loads per year, or 11 loads per year for a 50-ton Advanced Launch System (ALS). While U. S. launch capacities in the next decade or so are very uncertain and may well exceed this amount, it is hard to imagine significantly more launch capacity being devoted to SBI orbit maintenance.

The minimum practical altitude, whatever it is, cannot be lowered much by reducing satellite areas. The drag force  $D$  on a satellite of area  $A$  is roughly

$$D \propto \exp\left(\frac{-a}{h_0}\right) , \quad (7)$$

where  $a$  is orbit altitude and  $h_0$  is the atmospheric density scale height, about 7 or 8 km. Thus, for a given drag force, an order of magnitude reduction in satellite area allows

altitudes to be lowered by 2 or 3 scale heights, only about 20 km.

Air drag is not the only factor that affects fuel consumption rates at lower latitudes. Fuel consumption rates also can depend strongly on the level of solar activity. Strong solar activity heats the lower atmosphere, which in turn increases densities above about 120 km. The different curves in Fig. 2 shows propellant consumption rates at five levels of solar activity. The extreme levels represented by the outer curves probably do not persist long enough to have a big impact on long-term fuel requirements. (That is, the three inner curves are probably more indicative of the real range of long-term fuel requirements.) The uncertainty in fuel requirements due to solar activity variations is greatest at altitudes where the orbit maintenance requirements are smallest. For example, at 500 km, the three inner curves show an order of magnitude variation in fuel requirements, but the maximum is only 10 kg per year.

We examined the possibility that air drag could be reduced by using elliptical orbits with low-altitude perigees, instead of circular orbits entirely at low altitudes. To obtain the higher participation fractions associated with low altitudes, however, the orbits would have to be oriented correctly. (For inclined orbits, perigee should be near where the target is based.)

Maintaining proper orientation of the orbit would require correcting for a number of effects caused by the Earth's asphericity, chiefly rotation of the line of apsides. The rate of rotation is given by<sup>9</sup>

$$\Delta\omega = 6\pi J_2 \left[ \frac{R_\oplus}{a(1-e^2)} \right]^2 \left( 1 - \frac{5}{4} \sin^2 i \right) \text{ rad/rev} , \quad (8)$$

where  $J_2$  is the second-order coefficient in the Legendre polynomial expansion of the Earth's gravitational potential;  $R$  is the Earth's radius; and  $a$ ,  $e$ , and  $i$  are the semimajor axis, eccentricity, and inclination, respectively, of the orbit in question.

Correcting for apsidal rotation is done most efficiently by giving the satellite a velocity increment when it is at a point where its current orbit crosses the desired orbit.<sup>10</sup> There are two such points, and the correction may be made at either one. Figure 3 illustrates the geometry of correcting for apsidal rotation. It follows from basic orbital mechanics that the corrective velocity increment is

$$\Delta v = 2e \left[ \frac{GM}{a(1-e^2)} \right]^{1/2} \times \sin \left( \frac{\Delta\omega}{2} \right) , \quad (9)$$

where  $G$  is the universal gravitational constant and  $M$  is the Earth's mass. For small  $\Delta\omega$  (i.e., frequent corrections), this can be approximated by

$$\Delta v = e \left[ \frac{GM}{a(1-e^2)} \right]^{1/2} \Delta\omega . \quad (10)$$

Unlike corrections for drag, corrections for apsidal rotation are proportional to satellite mass and are independent of area. These rates can be obtained from Eqs.

(8) and (10) and are shown in Fig. 4 as a function of orbit inclination. No correction is required for  $i = 63.4^\circ$ , but consumption rates increase to nearly their maximum values at inclinations that differ by more than a few degrees.

For most inclinations, circular or very nearly circular orbits require the least total maintenance; this is true for a wide range of satellite properties. We conclude this by looking at combined (drag plus rotation) fuel requirements for orbits having a fixed perigee and different apogees. The apogee requiring the least total fuel is the best, at least from the point of view of orbit maintenance. Figure 5 is a plot of combined fuel consumption rates for a perigee at 200 km and apogees from 200 to 1200 km. We assumed an orbit inclination of  $53^\circ$ , which we show later is roughly optimal for targets at a  $50^\circ$  latitude (a latitude of interest). Because the proportion of maintenance fuel used to correct for rotation increases with orbit eccentricity, and because the drag and rotation components of the total fuel requirement depend differently on satellite mass and area, the eccentricity that requires the least total fuel depends on satellite mass and area. Figure 6 considers satellites of fixed mass (5000 kg) and different areas; Fig. 7 is the same type of plot for a fixed area and various masses. These figures show that in general, circular orbits require the least total maintenance. Slightly elliptical orbits would be desirable only for satellites that are the most inefficient aerodynamically and that consume the most fuel.

However, the conclusion that circular orbits are best does not hold for all orbital inclinations. This is shown in Fig. 8, which is a plot of total fuel consumption vs apogee altitude for a number of different inclinations. If inclinations within a few degrees of  $63.4^\circ$  can be used, fuel consumption would be minimized by using elliptical orbits. However, these inclinations could not be widely used. We

show below that defense performance depends very much on orbit inclination relative to target latitude; hence, inclinations near  $63.4^\circ$  would be useful only for targets very near about  $60^\circ$  latitude. While there may be targets near this latitude, the wide range in possible target latitudes (especially of SLBMs and mobile ICBMs) would not allow the defender to base many SBIs near  $63.4^\circ$ .

Our conclusions about orbit maintenance are as follows:

- (1) Orbit maintenance against air drag is a significant problem for satellites below about 350 km altitude, and probably precludes long-term basing of a large number of satellites below about 200 km.
- (2) In general, fuel consumption rates for orbit maintenance cannot be reduced by using elliptical orbits because additional corrections for apsidal rotation are required and result in even higher total fuel consumption.
- (3) The exception to conclusion (2) for orbits inclined within a few degrees of  $63.4^\circ$  is probably not important because these inclinations are useful against a very limited set of targets.
- (4) The above conclusions hold for a wide range of satellite masses and cross-sectional areas and a wide range of solar-activity levels.

### **III. ORBIT ALTITUDE OPTIMIZATION**

In this section, we explain a quantitative method for comparing the benefits of low-altitude orbits (improved participation) to the penalties (more air drag) and find optimum orbit altitudes for various threats. Our approach is

to find the altitude that gets the best performance out of a given mass of rocket propellant, which is used either for orbit maintenance (at lower altitudes) or to improve SBI performance (at higher altitudes). We assume a fixed combined mass for SBIs and orbit maintenance fuel; as altitudes are lowered and more fuel is needed to offset air drag, the SBIs have correspondingly less mass, velocity, and range.

This trade-off is explained in detail in Part 1 below. Part 2 of this section explains how we measure defense effectiveness, and in Part 3 we find optimum altitudes for circular orbits.

#### 1. PROPELLANT ALLOTMENT TRADE-OFFS

The mass of each SBI depends on the orbit maintenance fuel consumption rate. SBI masses are found by assuming that each carrier vehicle has a fixed payload mass (usually 3000 kg). This fixed payload mass includes 10 SBIs and fuel needed for 1 yr of orbit maintenance. Therefore, for each orbit considered, the total mass of all SBIs is the CV payload mass minus the mass of fuel needed to maintain the orbit for 1 yr.

The maximum velocity of each SBI is determined from its mass. Our idealized SBI consists of a payload and a three-stage rocket. The payload mass is 5 kg (Ref. 11) unless otherwise specified and does not depend on either the properties of the orbit or of the rest of the SBI. The rocket is designed to achieve the highest possible velocity from its fuel allotment. This is done by giving each stage an equal share (one third) of the total SBI velocity.<sup>7</sup> That velocity is calculated using the following assumptions:



1. Each stage obeys the single-stage rocket equation [Eq. (5)].
2. Each stage has a 10% structural-mass fraction.
3. All propellants have a specific impulse of 300 sec.

The maximum velocity of the SBI increases rapidly with mass up to about 100 kg (5- to 10-km/sec velocity, depending on payload mass), but increases slowly thereafter. This is shown in Fig. 9. We will see that, combined with the steep dependence of air drag on altitude, the steep mass-velocity dependence means that SBI velocities will decrease very rapidly as orbits are lowered.

## 2. DEFENSE PARTICIPATION FRACTIONS

Our measure of the effectiveness of space-based defenses is the DPF, which is the fraction of weapons in orbit that can attack at least one target in the time available (e.g., the booster burnout time). The inverse of the DPF is sometimes called the absentee ratio.

We calculate the DPF analytically in the limit of large numbers of weapon-carrying platforms, as explained in Appendix A. This treatment ignores statistical fluctuations in the number of participating platforms that occur because the locations of the platforms in their orbits at the time of attack cannot be controlled by the defender. These fluctuations can be significant for defenses with few weapon platforms.

Most participation fractions in this study are lower than would be seen in practice because, for simplicity, we generally consider threats based at a single geographical point; dispersed threats would allow significantly more participation. However, as we show

later, the dependence of DPF on altitude does not change much with threat geography, and assumptions about geography thus have no significant effect on our results.

Lower orbit altitudes can significantly improve defense performance (as measured by the DPF), but only against threats where higher-altitude defenses perform poorly or not at all. This is shown in Fig. 10, where air drag is ignored and SBI velocity is 6 km/sec in all cases.

(Throughout this study we assume that threat burnout altitude in kilometers equals burnout time in seconds.<sup>1</sup>) We see that reducing orbit altitudes helps the defense significantly only against threats that otherwise allow little or no defense participation.

We can now show why elliptical orbits are not generally useful. Figure 11 shows that defense participation is very sensitive to orbit inclination relative to target latitude. Thus, while elliptical orbits near  $63.4^\circ$  inclination require less maintenance than circular orbits, they can be used to attack only a very limited latitude range (about  $10^\circ$  if the SBI range is about 1000 km).

If fuel for drag compensation is obtained at the expense of SBI velocity, as explained above, defense performance eventually gets worse as altitudes are lowered (Fig. 12). The steepness of this decline is due to the rapid increase in drag at lower altitudes plus the steep dependence of SBI velocity on mass in this regime.

### 3. ORBIT ALTITUDE OPTIMIZATION

We define optimum altitude as that which requires the least total fuel (for SBIs plus maintenance) per SBI in

the battle. That is, our measure of orbit merit is lifecycle mass of fuel per SBI divided by the DPF. Lifecycle fuel mass per SBI is the mass of fuel in each SBI plus its share of orbit maintenance fuel, which is

$$\text{maintenance fuel (kg)} = \frac{\text{SBI life (yr)} \times \text{CV fuel consumption (kg/yr)}}{\text{Number of SBIs per CV}}.$$

We assume 10 SBIs per CV and, unless stated otherwise, a 10-yr satellite life.

Even quite responsive threats do not call for low-altitude basing. For SBI flight times as low as about 90 sec, our approach indicates that altitudes above about 400 km are the most fuel-efficient. This is shown in Fig. 13, where we assume a 60 sec defense start-up time (i.e., SBI time-of-flight is 60-sec less than threat burnout time). Also, altitudes can be raised significantly above optimum (for survivability) with very little performance penalty.

SBI defenses become impractical if SBI flight times are less than about 50 sec. This situation would demand a combination of very low orbits and very fast SBIs. For example, Fig. 13 shows that for a threat that burns out in 100 sec at 100 km, an orbit altitude of about 220 km is optimum if the defense start-up time is 60 sec. As argued above, this is probably about as low an orbit as can be maintained, even if significant national resources are devoted to the problem. Moreover, even at the optimum altitude and velocity, lifetime propellant requirements per intercept against these threats are enormous (on the order of  $1 \times 10^7$  kg, about 200 ALS flights). Since several hundred intercepts may be

required, this mission strains the imagination, even though our participation fractions are somewhat low.

In general, these conclusions are not sensitive to our assumptions. Some examples of this are shown in Figs. 14 through 18 for a 150-sec burnout-time threat. Optimum altitudes can be somewhat lowered by drastically reducing satellite frontal area (Fig. 15). However, the figure also shows that this is not much more fuel-efficient than basing less aerodynamic satellites at higher altitudes. Defense lifetimes much shorter than our 10-year nominal value would also lower optimum altitudes (Fig. 16). This would be much more expensive and not much more fuel-efficient than longer-term basing at higher altitudes, however. CV payload mass (Fig. 17) and longitudinal extent of threat basing (Fig. 18) have very little impact on altitude optimization.

#### **IV. CONCLUSIONS**

Low-altitude basing of SBIs is a possible counter-response to faster-acting ICBMs. We have quantified some of the advantages and disadvantages of this idea and identified optimum altitudes against various threats. Survivability was not a factor in this optimization. Our main findings are

- (1) Performance of SBI defenses is not generally sensitive to orbit altitude. Only for highly responsive threats, which reduce SBI participation to zero or very near zero, do lower altitudes improve performance by large factors.
- (2) Air drag becomes an important problem below about 350 km and is very sensitive to altitude. Long-term basing below 200 km is probably impossible for large constellations.

- (3) Orbit maintenance cannot generally be reduced by using elliptical orbits. Reduced maintenance for drag is more than offset by necessary compensation for apsidal rotation.
- (4) Even against responsive threats (150-sec burnout time), our approach recommends orbits above about 400 km altitude. Significantly faster threats (less than about 100 sec) are extremely difficult to attack with SBIs at any altitude, assuming a 60-sec launch delay.

## **ACKNOWLEDGEMENTS**

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## Appendix A: Calculation of Defense Participation Fractions

Our expression for the participation fraction is

$$F = \left(\frac{1}{\pi}\right) \left(\frac{dp}{dv}\right) (2\phi_{\max} + \Delta\phi) dv \quad .$$

Here  $v$  is the angular position, relative to perigee, of the platform in its orbit, measured at the center of the Earth. It is related to platform latitude by the equation

$$\sin(l) = \sin(i) \cos(v) \quad .$$

The factor  $dp/dv$  measures the density of platforms in a given orbit; i.e.,

$$\frac{dp}{dv} = 1 \quad .$$

If platforms are distributed "evenly" in the orbit (i.e., so that they pass a given point in the orbit at equal time intervals) then

$$\frac{dp}{dv} = \frac{(1-e^2)^{3/2}}{2\pi(1+e \cos v)} \quad .$$

The quantity  $2 \phi_{\max}$  is the range in longitude over which a platform at latitude  $l$  could participate in the battle. It is related to the maximum angular range  $\gamma_{\max}$  of the SBI by

$$\cos(\gamma_{\max}) = \cos(l) \cos(l_t) \cos(\phi_{\max}) + \sin(l) \sin(l_t) ,$$

where  $l_t$  is the target latitude. The angular range  $\gamma_{\max}$  is measured at the center of the earth; it is found from the properties of the SBI and the properties of its orbit using the equation

$$\cos(\gamma_{\max}) = \frac{R_p^2 + R_t^2 - D_c^2}{2R_p R_t} .$$

Here  $R_p$  and  $R_t$  are the Earth-center distances of the platform (at time of SBI launch) and of the target at time of intercept. The term  $D_c$  is the maximum range of the SBI. We calculate this assuming that the SBI accelerates at 20 g up to its maximum velocity, and that the SBI's time of flight is the target lifetime (e.g., burnout time) minus the defense start-up time (the time between ICBM launch and SBI launch).

Our expression for the DPF can treat targets spread out in longitude, but not in latitude. The final term in the integrand for  $F$ ,  $\Delta\phi$ , is the longitudinal size of the target set (unless  $\phi_{\max}$  is zero, in which case  $\Delta\phi$  and  $F$  are also zero).

The DPF depends on many parameters. The table below lists default parameter values we use to calculate DPFs.



Table A-1. Default values in DPF calculations

Variable	Parameter	Value
Threat	Latitude	50°
	Longitudinal size	0°
	Burnout time	150 sec
	Burnout altitude	150 km
SBI	Launch delay	60 sec
	Acceleration	20 g
Orbit	Altitude	500 km
	Inclination	52°
	Eccentricity	0

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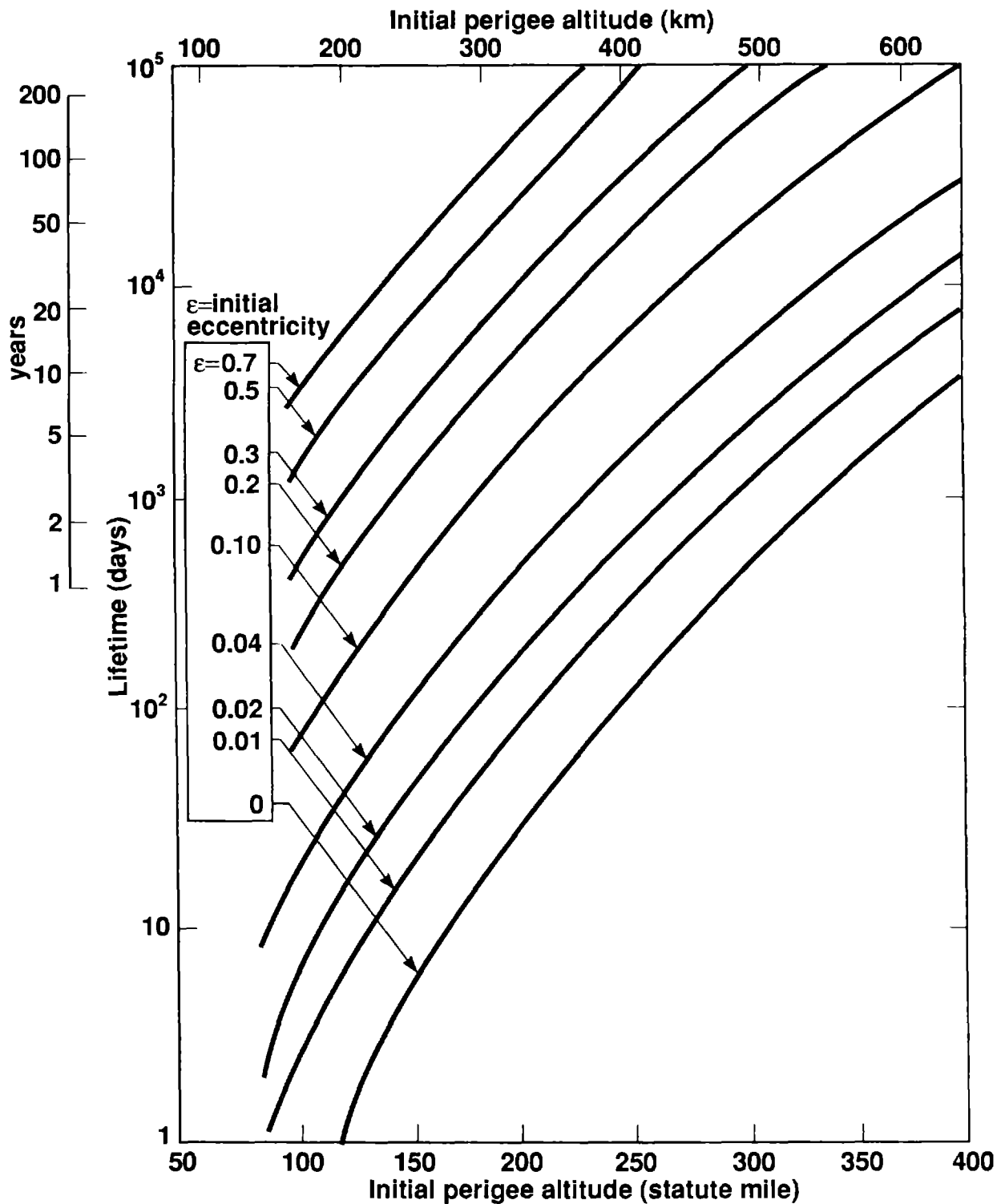


Figure 1. Lifetimes of satellites in elliptical orbits as a function of orbit eccentricity and altitude at perigee.<sup>12</sup> Lifetimes are unacceptably short unless orbits are maintained. The assumed value of satellite  $\beta$  ( $\frac{M}{C_d A} = 80 \text{ kg/m}^2$ ) is typical of previous satellites. Our baseline assumption ( $\beta = 250 \text{ kg/m}^2$ ) would allow lifetimes about 3 times longer than those shown here.

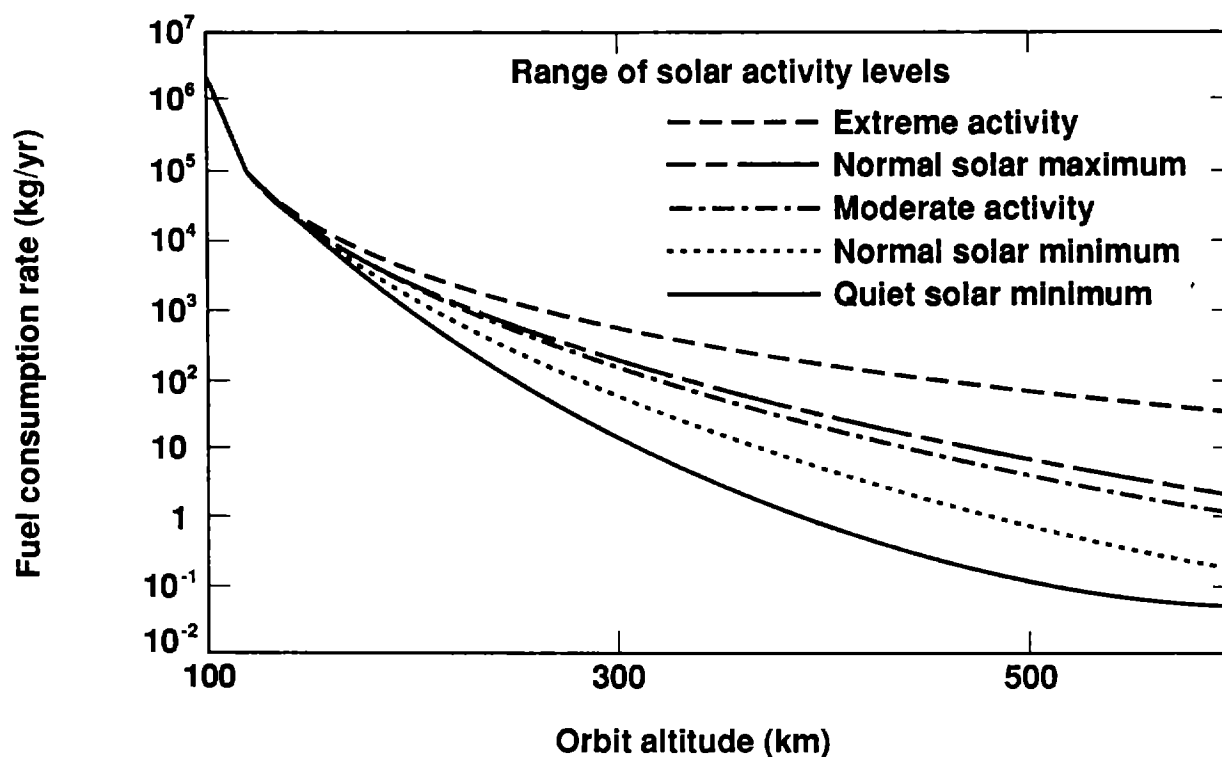


Figure 2. Stationkeeping fuel needed per year to overcome drag in circular orbits. Fuel requirements are very different at different levels of solar activity. The various curves represent atmospheric densities<sup>8</sup> corresponding to solar activity levels ranging from a very quiet solar minimum to extreme conditions at solar maximum.

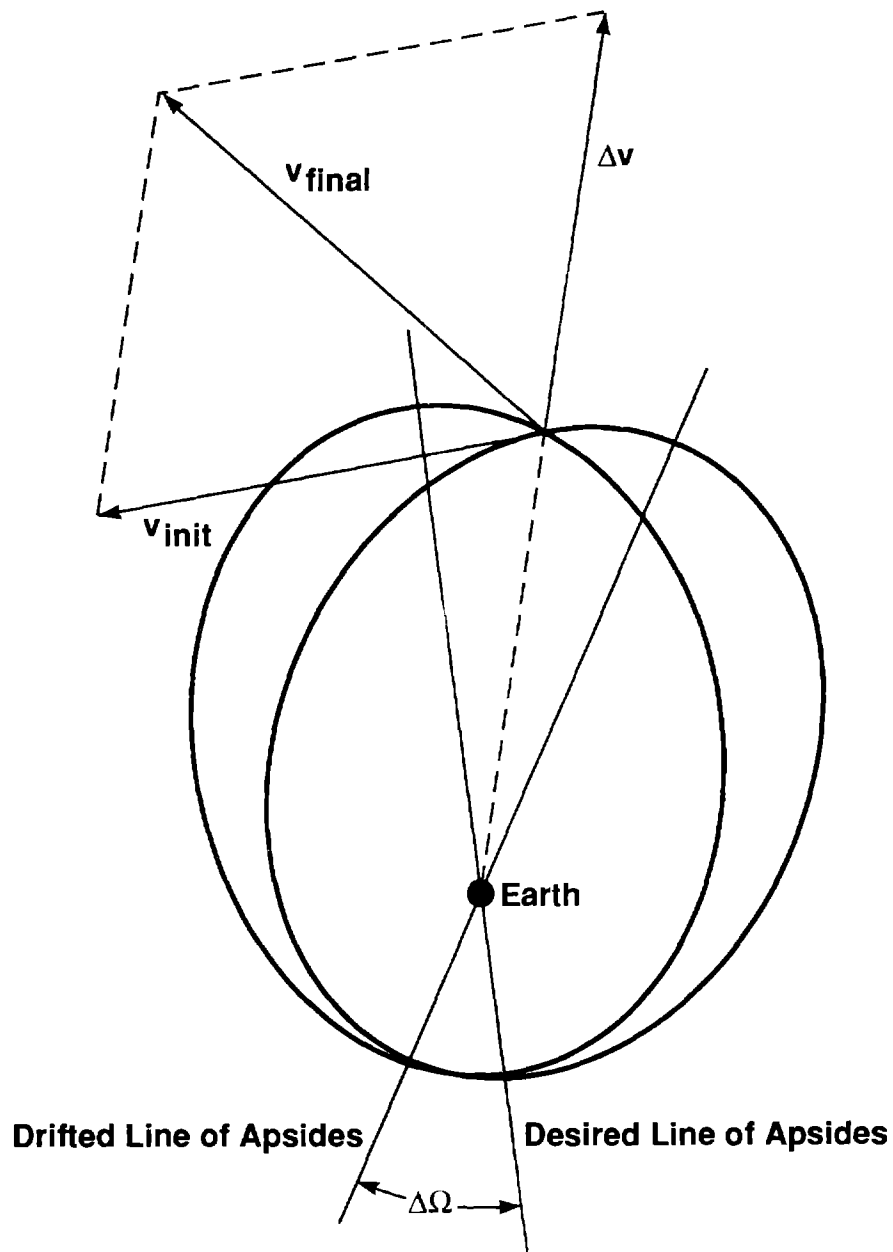


Figure 3. Orbit geometry for efficient apsidal rotation corrections. A thrust of magnitude  $\Delta v$  is applied when the satellite crosses the path of the desired orbit. Figure is not to scale.

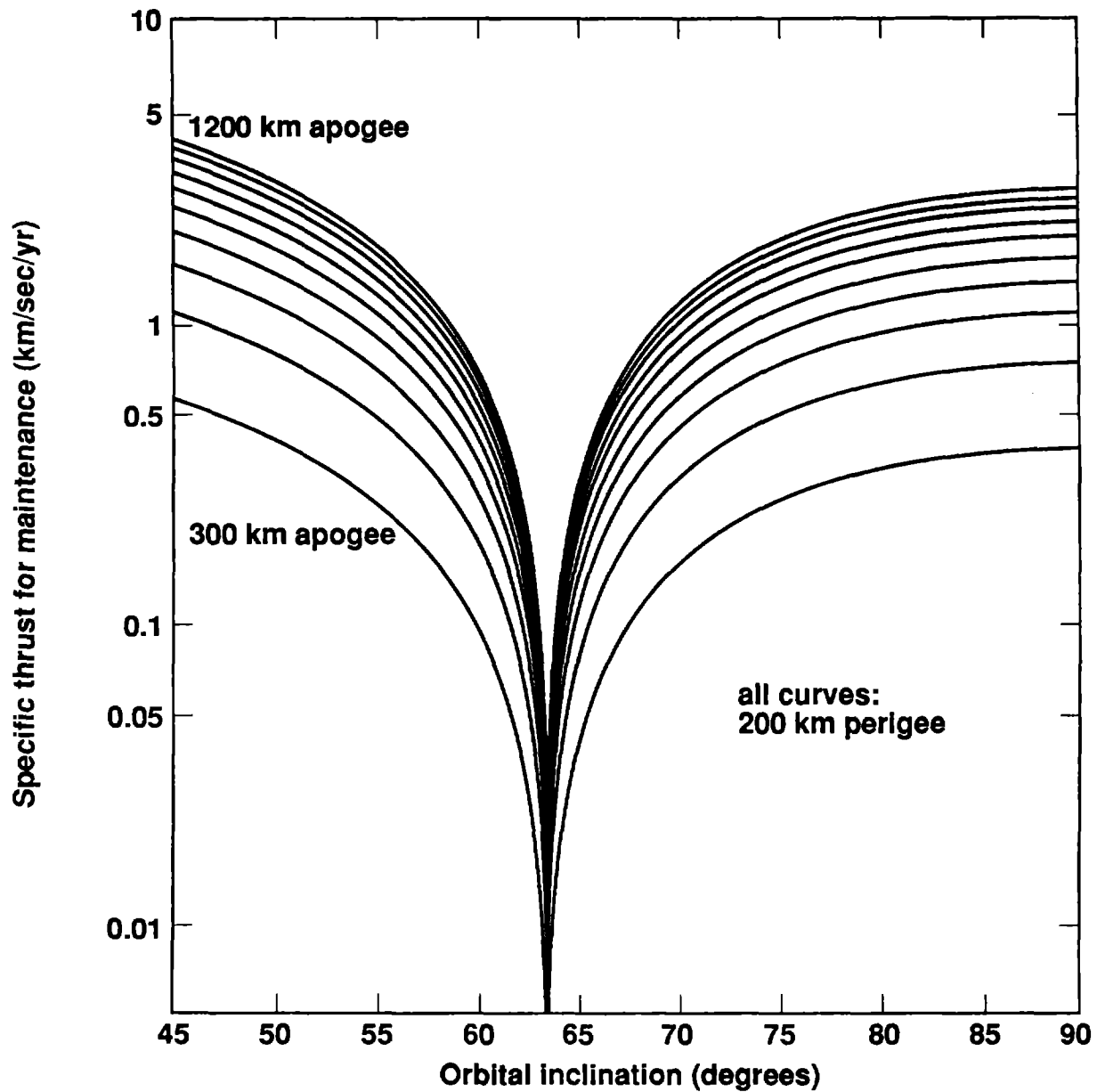


Figure 4. Specific thrust (applied velocity) needed to correct for apsidal rotation of elliptical orbits having a 200-km altitude at perigee. Different curves correspond to apogee altitudes from 300 to 1200 km. No correction is needed for orbits inclined  $63.4^\circ$  relative to the equator.

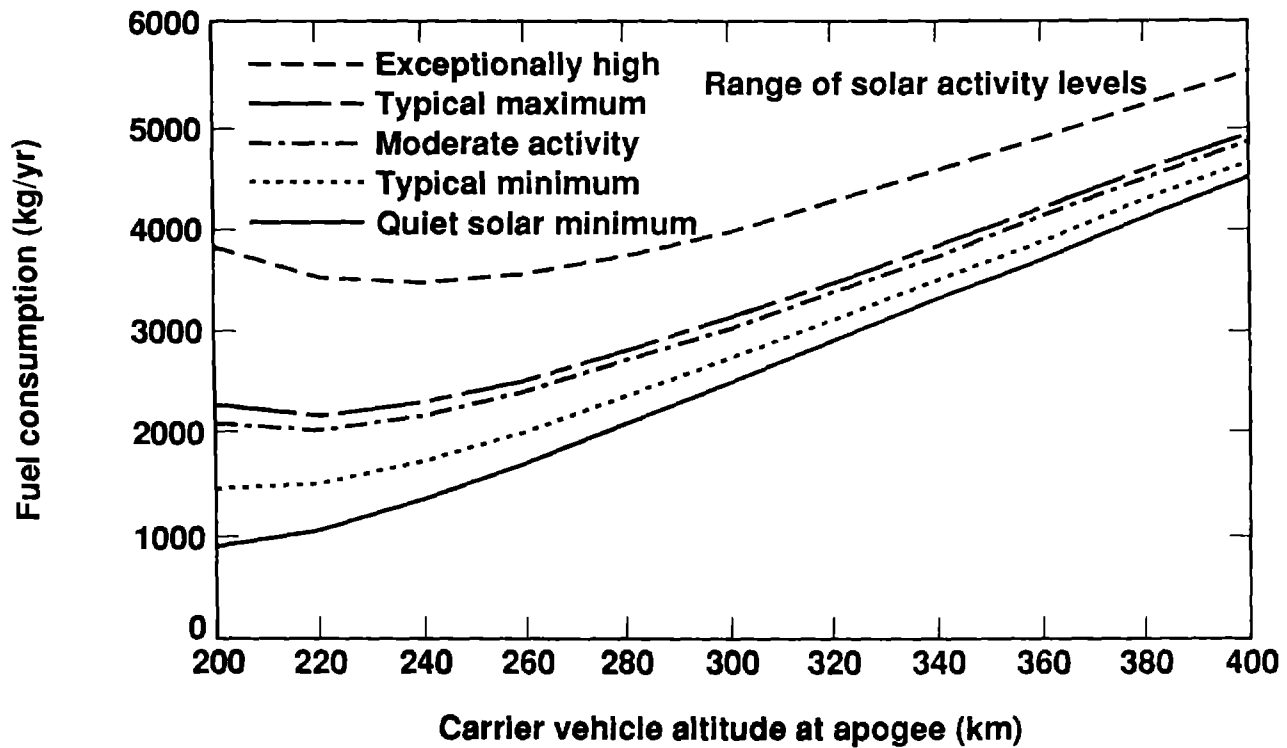


Figure 5. Combined fuel consumption (atmospheric drag plus apsidal rotation) for satellites in elliptical orbits. Altitude at perigee is 200 km in all cases. Horizontal axis is altitude at apogee; thus orbit eccentricity increases towards the right of the plot. Fuel consumption is minimized by using circular or very nearly circular orbits. Different curves correspond to the solar activity levels shown in Fig. 2. Orbit inclination is 53°.



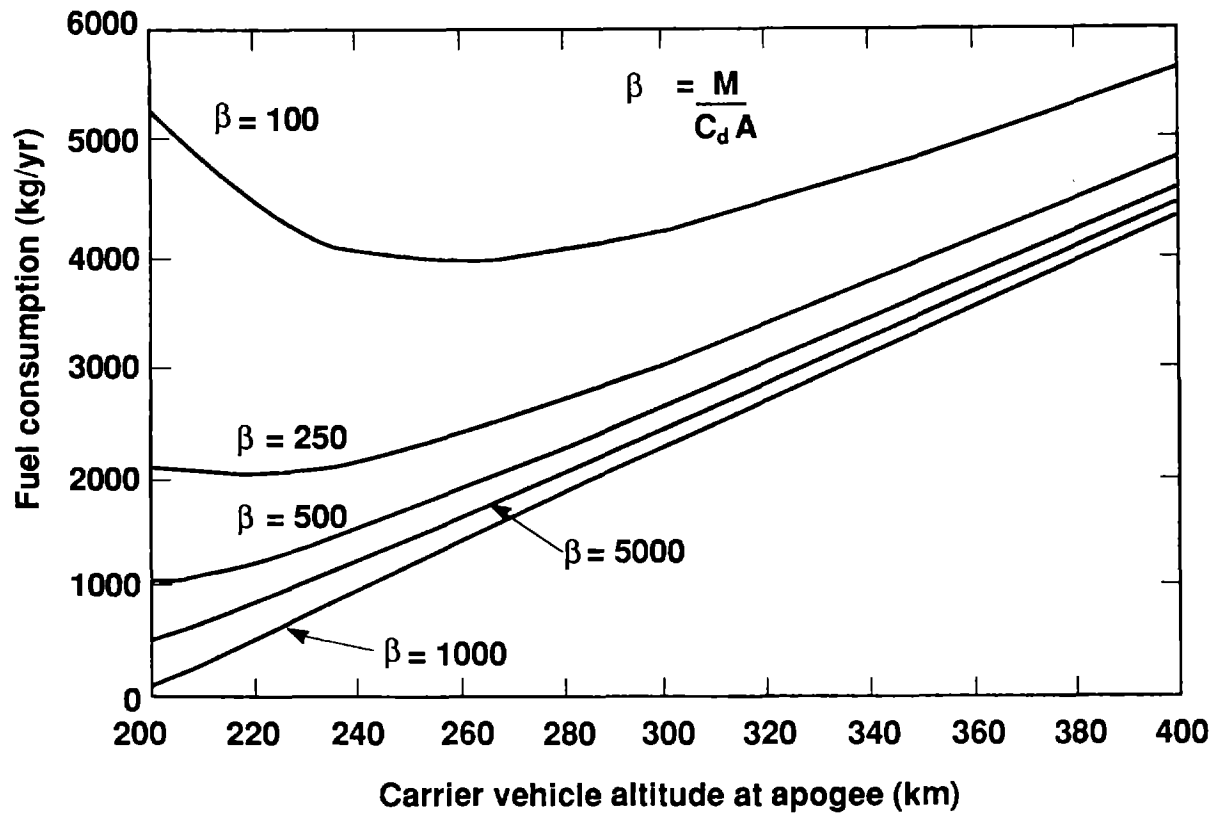


Figure 6. Combined fuel consumption (atmospheric drag plus apsidal rotation) for satellites in elliptical orbits. Altitude at perigee is 200 km in all cases. Horizontal axis is altitude at apogee. Different curves represent satellites of different betas ( $\beta = \frac{M}{C_d A}$ ). Again, fuel consumption is minimized with circular or nearly circular orbits. The satellite mass is 5000 kg and orbit inclination is  $53^\circ$  in all cases.

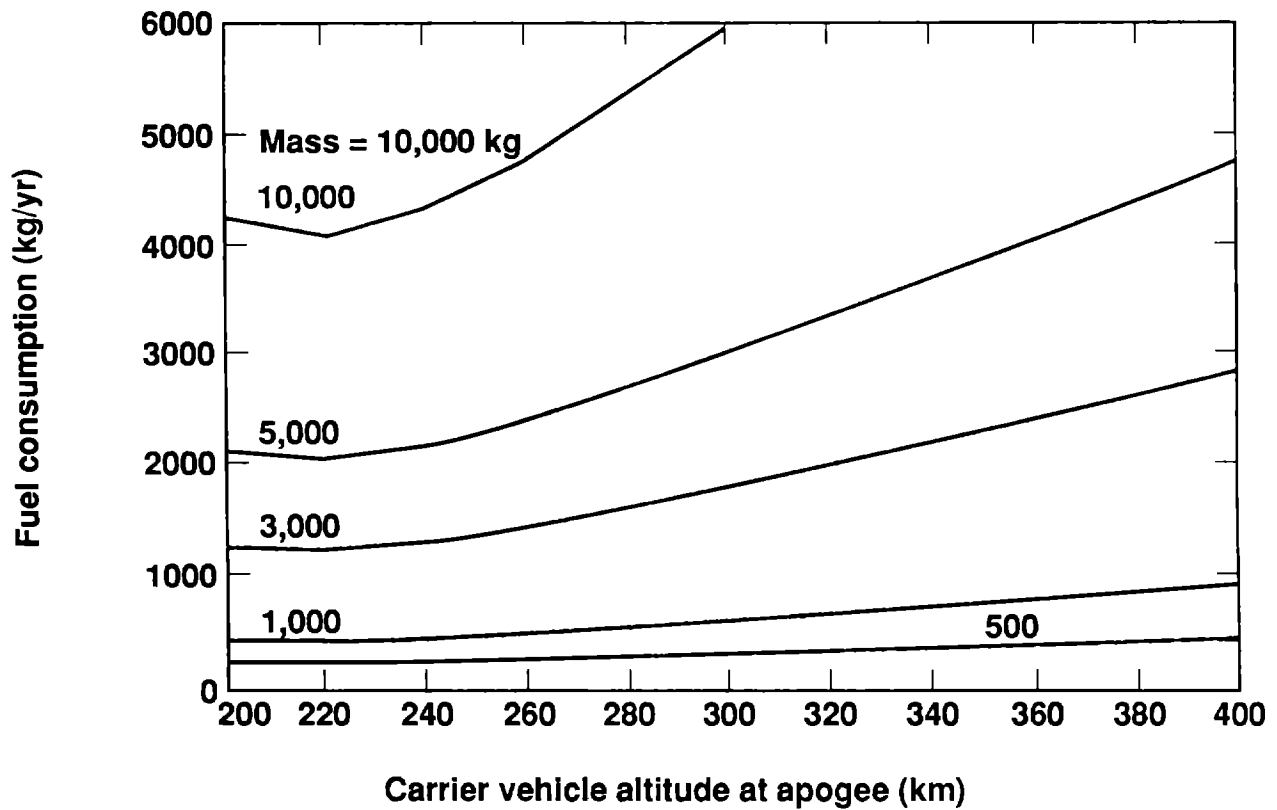


Figure 7. Combined fuel consumption (atmospheric drag plus apsidal rotation) for satellites in elliptical orbits. Horizontal axis is altitude at apogee; altitude at perigee is 200 km in all cases. Different curves represent satellites of different mass. In no case do elliptical orbits save much fuel compared to circular orbits. Satellite beta is 250 kg/m<sup>2</sup> in all cases.

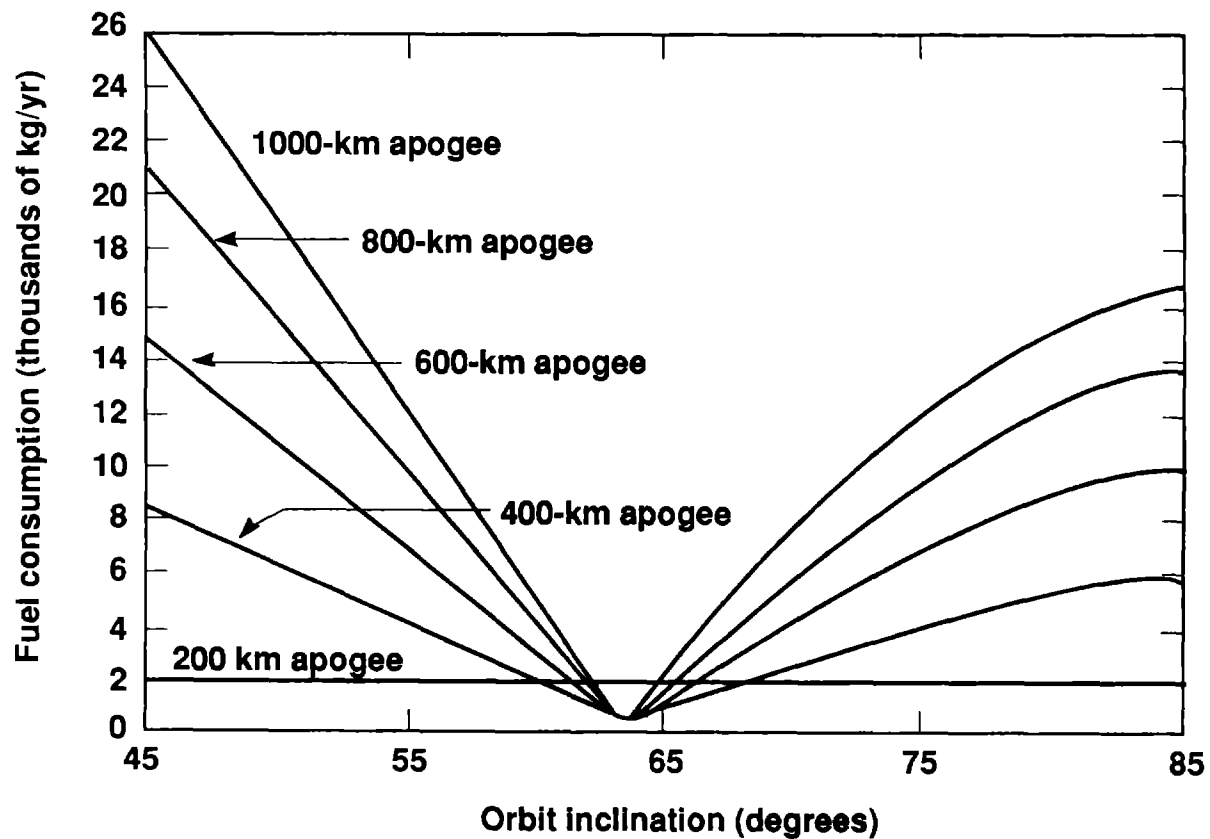


Figure 8. Combined fuel consumption (drag correction plus apsidal rotation correction) vs orbit inclination for satellites in elliptical orbits. Different curves represent different apogee altitudes; perigee is always at 200 km. Elliptical orbits use less fuel than circular orbits if inclinations are within a few degrees of  $63.4^\circ$ .

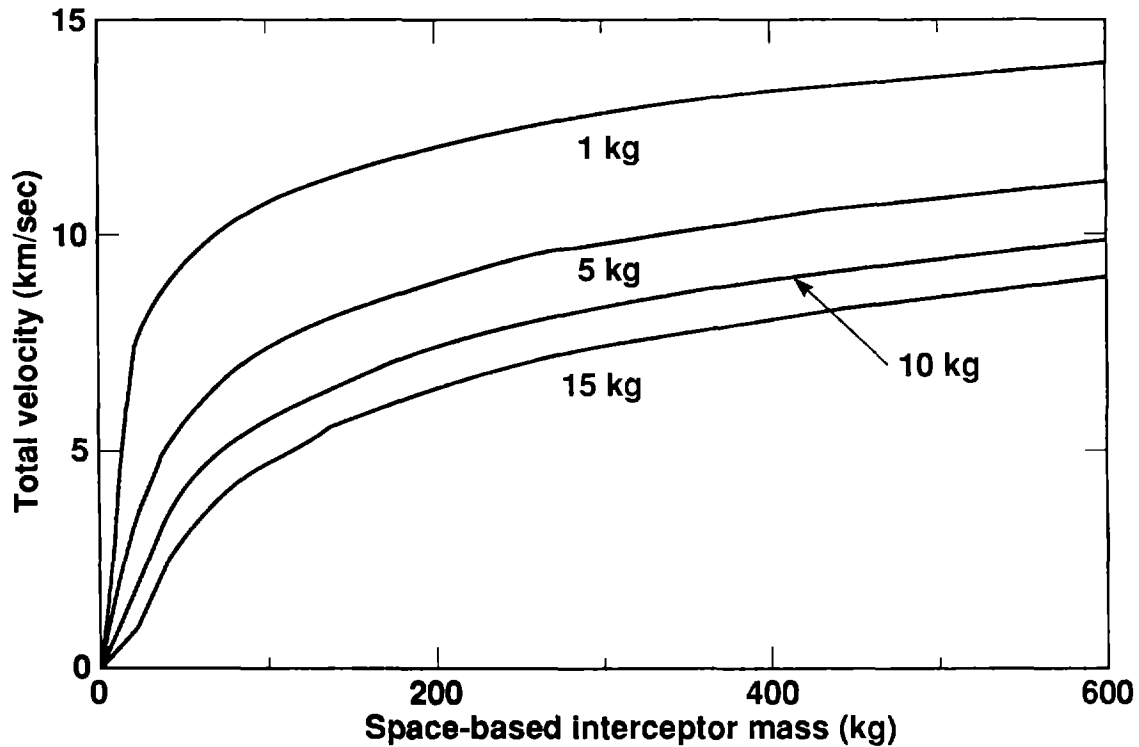


Figure 9. Maximum velocity of three-stage interceptor rockets vs rocket mass. Curves for several different payload masses are shown. These velocities are used in subsequent analyses. We assume a propellant specific impulse of 300 sec and ignore gravity. Each stage has a 10% structural-mass fraction, and each contributes equally to the total velocity. Actual interceptors may be less efficient; also they will have to use some of their velocity for steering.

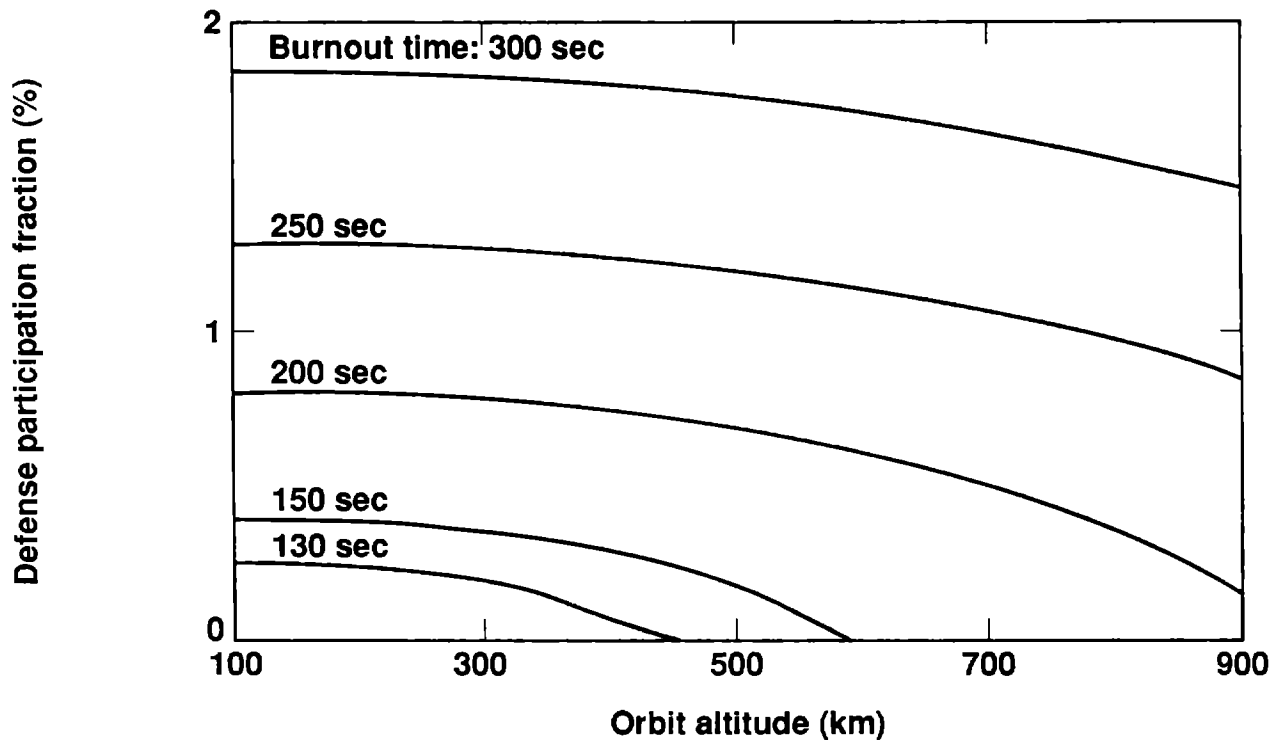


Figure 10. Defense participation fraction vs altitude for SBIs in circular orbits. Low altitudes increase the participation fraction only slightly. Threat burnout times are labeled, and threats are varied such that burnout altitude in km equals burnout time in seconds. SBIs are released 60 sec after the threat is launched; they have 20 g acceleration and 6 km/sec velocity in all cases. The threat is based at 50° north latitude and has no significant geographical extent (i.e., the threat represents one launch field). Dispersed threats would allow much higher participation. Orbit inclination is 52°, approximately optimum for the assumed threats.

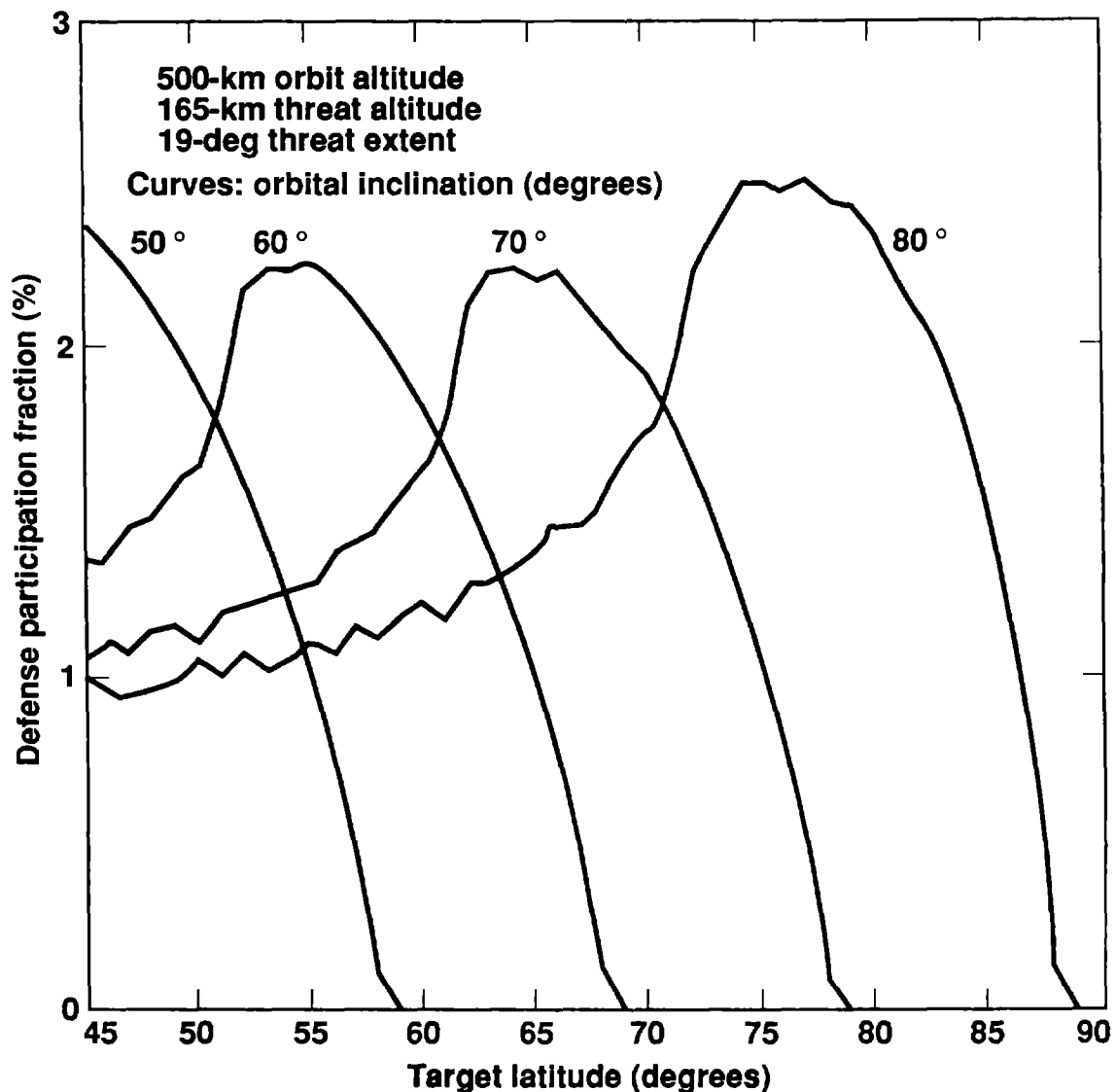


Figure 11. Defense participation fraction vs latitude of target basing. Different curves represent different orbital inclinations. Any given inclination is effective against threats in only a narrow range of latitudes. Here, the threat burns out at 165 km, and has an extent in longitude of 19°. SBI range is 1000 km, corresponding to, for example, a flight time of about 165 sec at 6 km/sec constant speed.

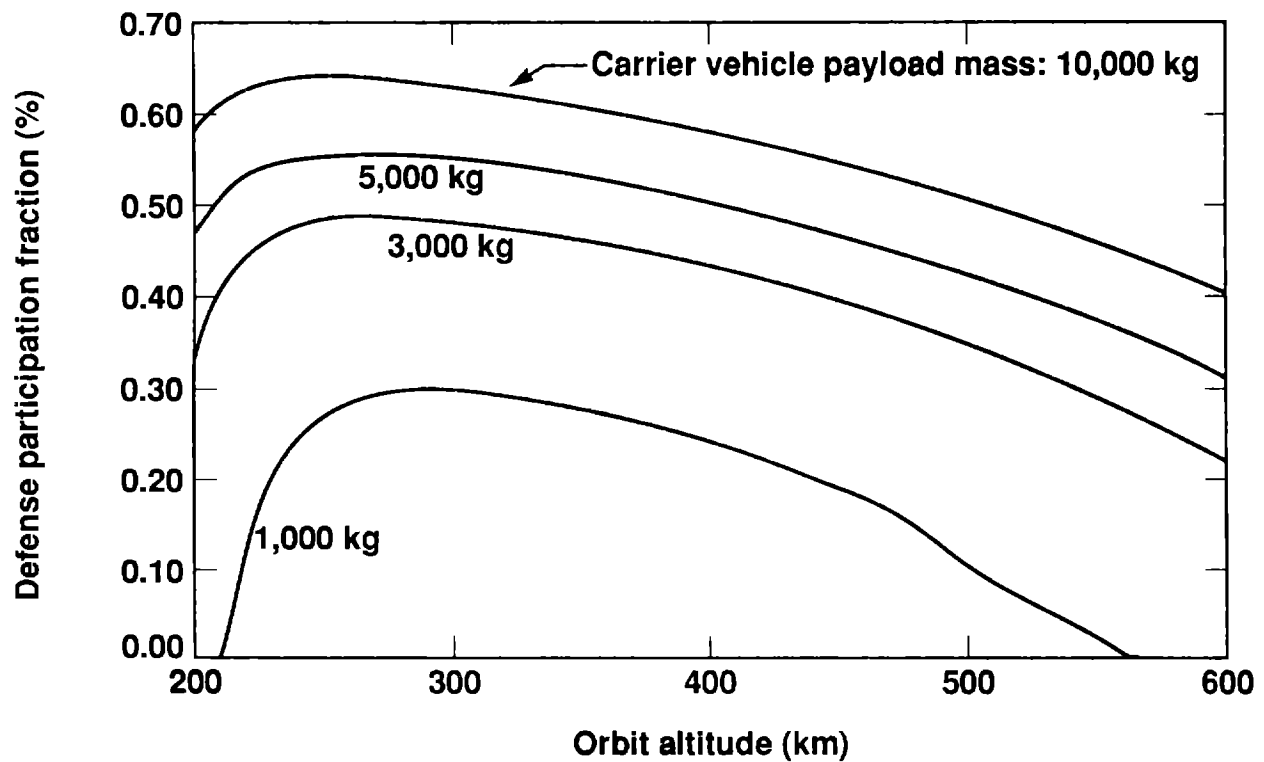


Figure 12. Defense participation fraction vs altitude. Propellant is traded between stationkeeping and SBIs, as explained in the text. Different curves correspond to different carrier vehicle payload masses, as labeled. Each payload consists of 10 SBIs and enough fuel to maintain the orbit for 1 yr. Performance drops off at low altitudes because stationkeeping demands most of the fuel allotment, and at high altitudes because fewer SBIs have enough range to reach the threat.

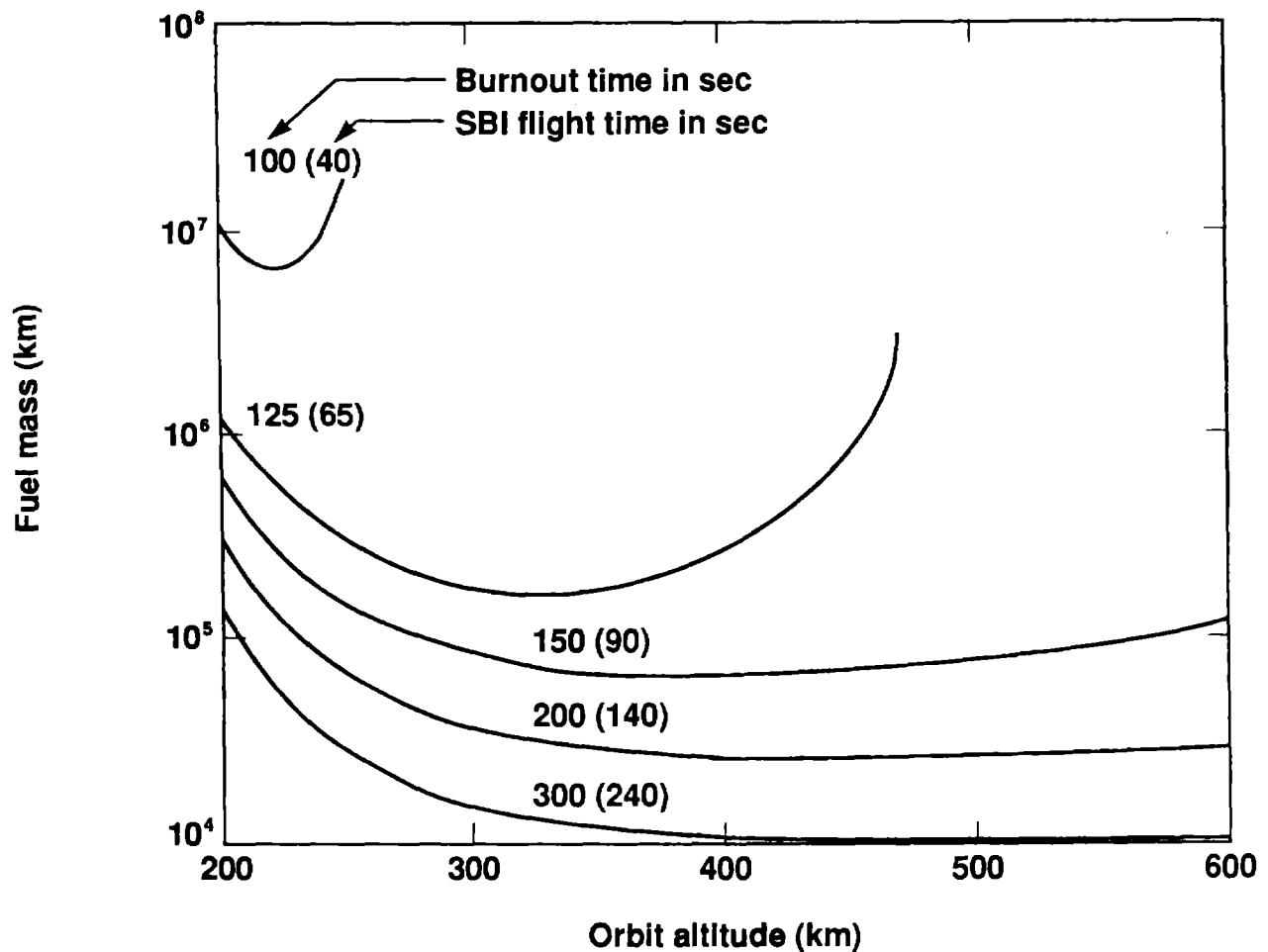


Figure 13. Altitude optimization for circular orbits. Orbit utility is measured by lifecycle fuel required per participating SBI (fuel in the SBI itself plus its share of orbit maintenance fuel, divided by defense participation fraction). This quantity is plotted as a function of orbit altitude. The different curves represent different threats (again threat burnout time in seconds equals burnout altitude in km). Except against very fast threats, where participation reactions are probably unacceptably low, optimum altitudes are above about 300 km and can be much higher than that. SBI flight times corresponding to a defense start-up time of 60 sec are shown in parentheses.



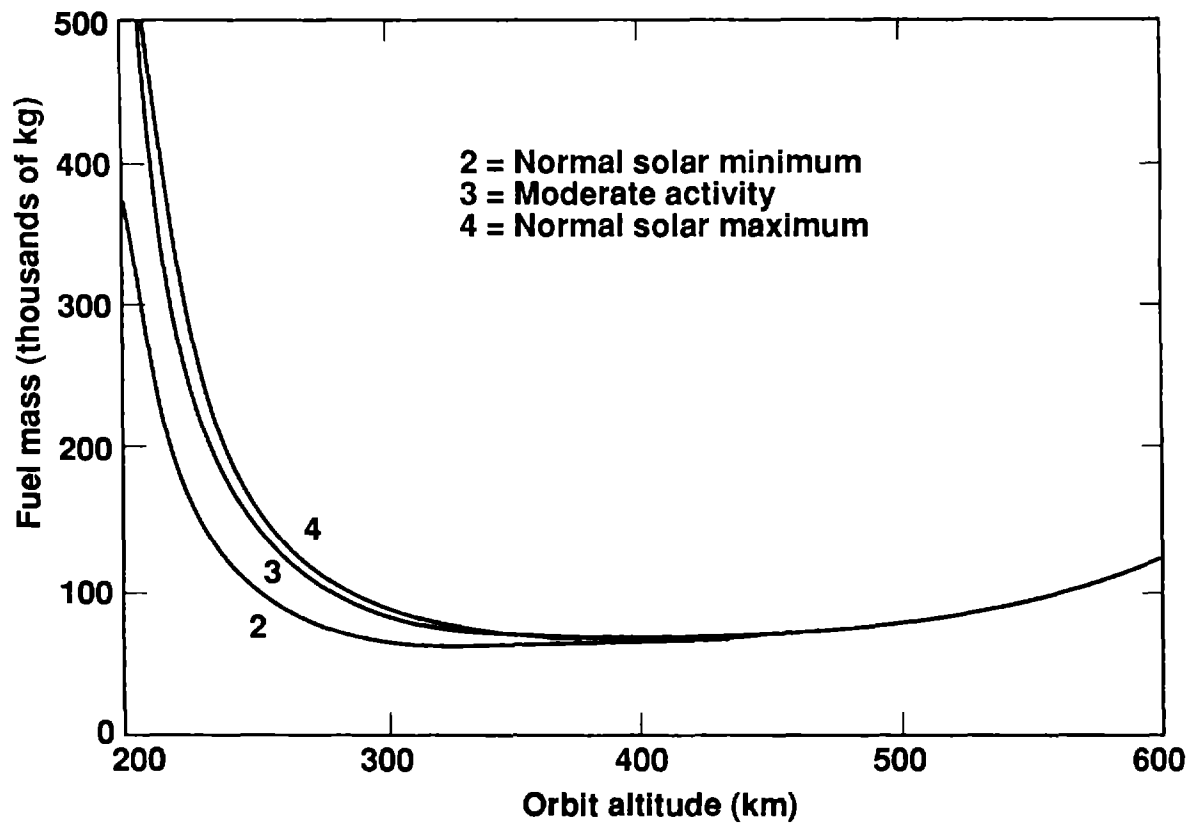


Figure 14. Required lifecycle fuel per participating SBI vs altitude of circular orbits. Different curves correspond to different levels of solar activity. Fuel consumption at optimum altitude (i.e., minimum fuel consumption) does not depend much on solar activity level. The threat burnout time (altitude) is 150 sec (km).

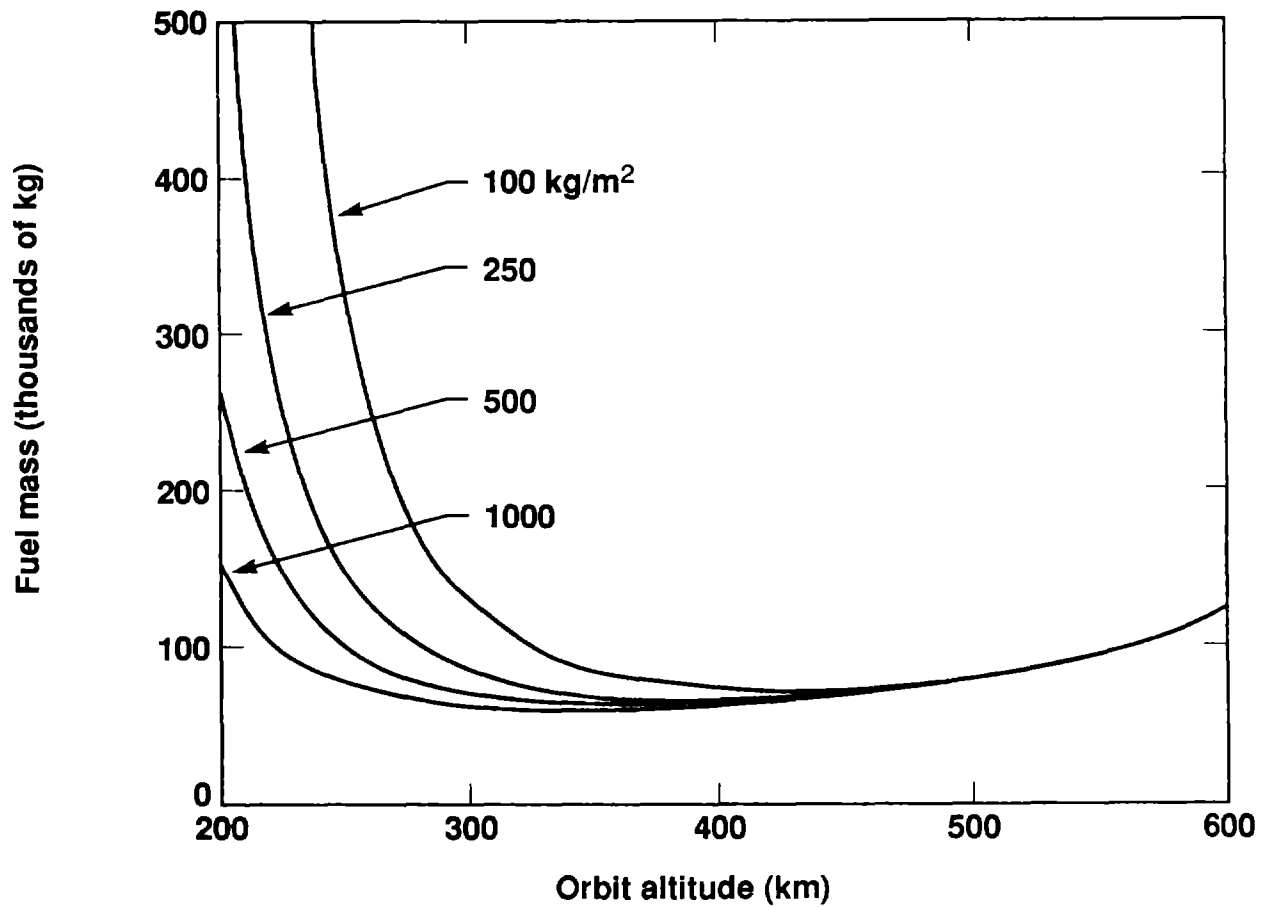


Figure 15. Lifecycle fuel mass required per participating SBI vs orbit altitude. Curves represent different values of satellite beta ( $\beta = \frac{M}{C_d A}$ ). Fuel required at optimum altitude (that which requires the least fuel) is about the same for all betas shown. The threat burnout time (altitude) is 150 sec (km).

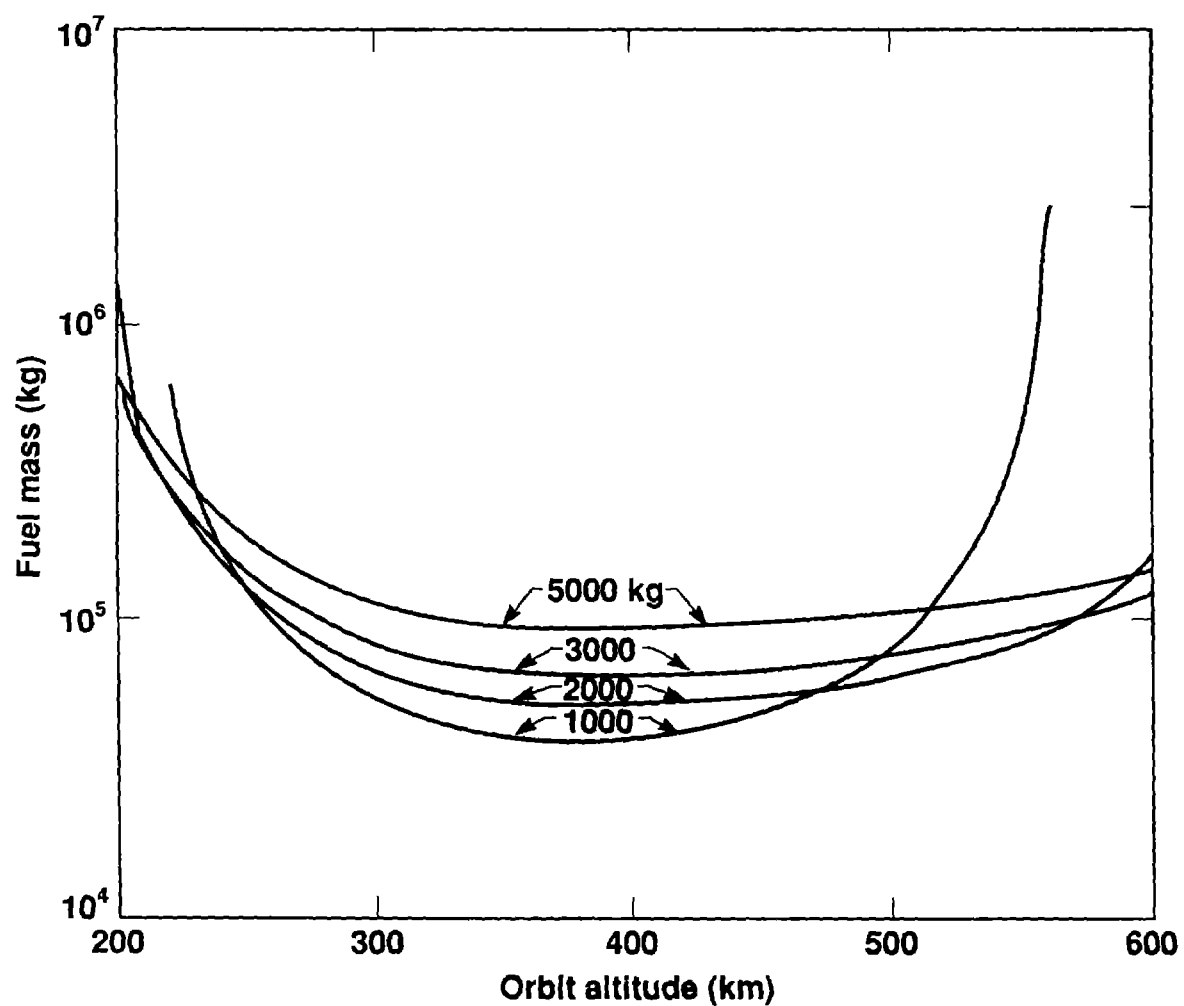


Figure 16. Lifecycle fuel per participating SBI vs orbit altitude. Curves correspond to different satellite masses. Optimum altitude (that which requires the least fuel) is the same for all masses shown. Satellite beta is  $250 \text{ kg/m}^2$  and threat burnout time (altitude) is 150 sec (km).

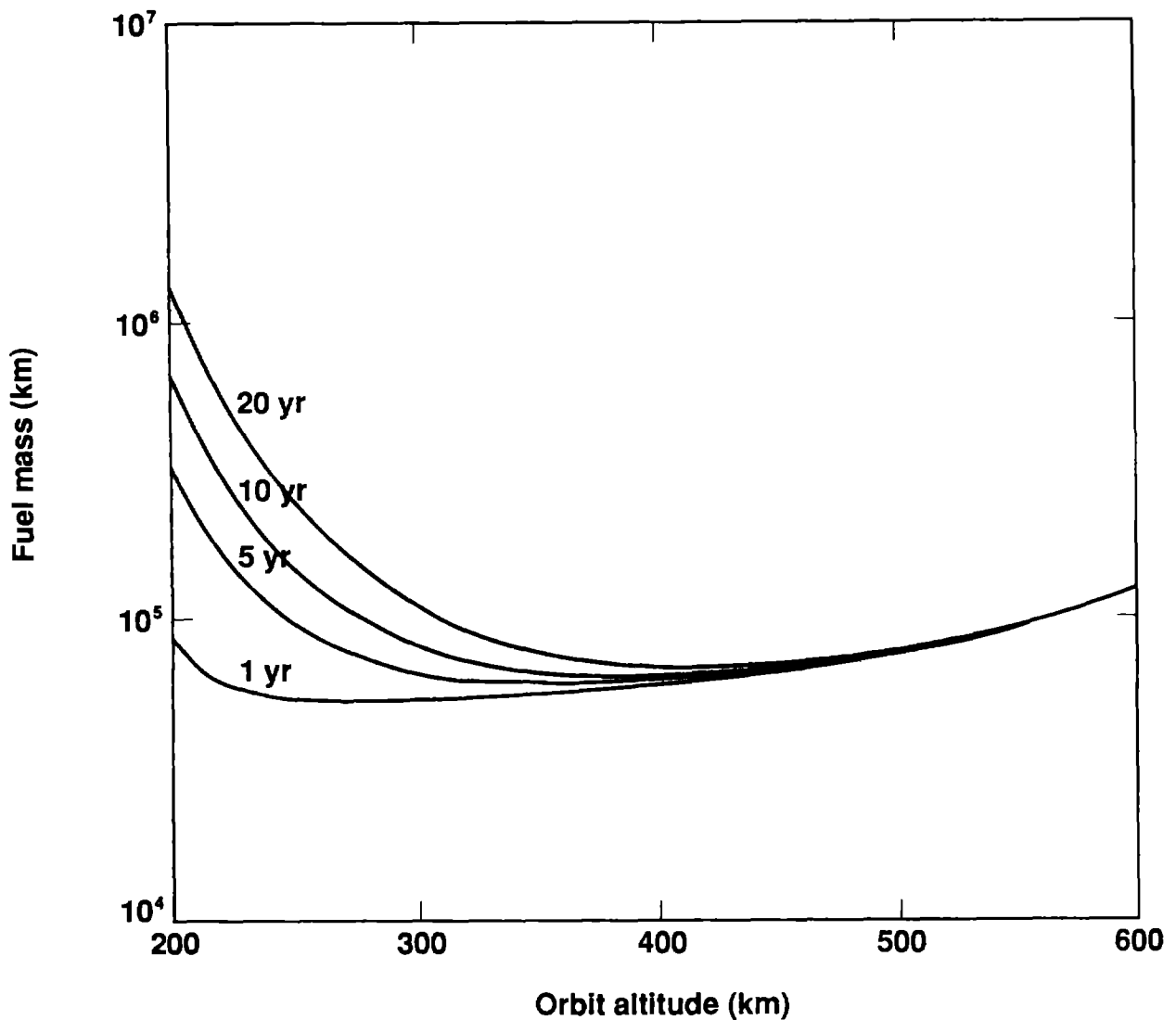


Figure 17. Lifecycle fuel required per participating SBI vs orbit altitude. Curves show dependence on assumed lifetime for the defense. Fuel consumption is minimized at lower altitudes for shorter-lived satellites because a smaller fraction of the fuel is used to maintain the orbits. (However, short-lived defenses are more expensive.) Satellite beta is  $250 \text{ kg/m}^2$  and threat burnout time (altitude) is  $150 \text{ sec (km)}$ .

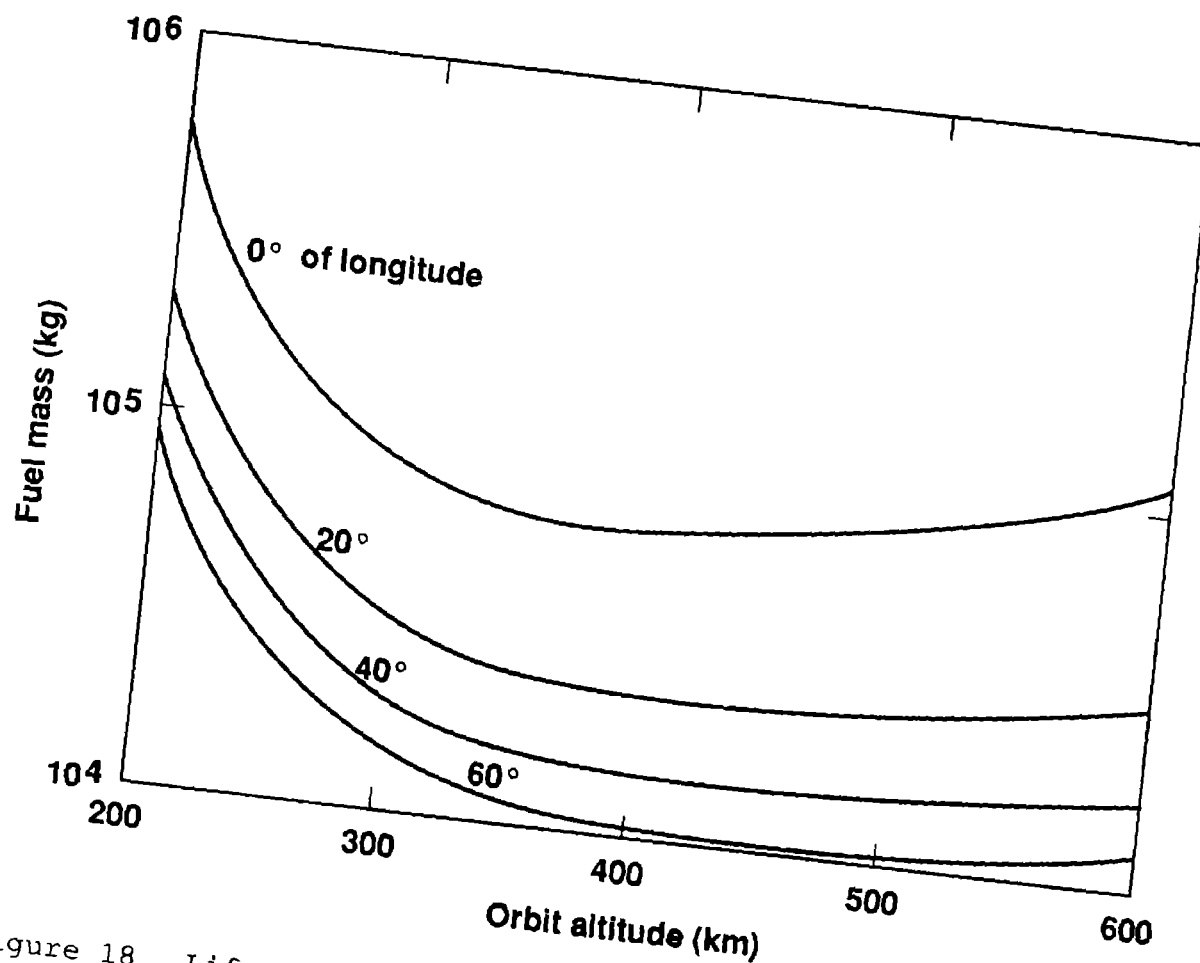


Figure 18. Lifecycle fuel required per participating SBI vs orbit altitude. Curves show dependence on extent of threat launch points in longitudinal direction. Optimum altitude is about 400 km regardless of threat extent. Threat burnout time (altitude) is 150 sec (km); satellite beta is 250 kg/m<sup>2</sup>.

